TransDatRO_code_source

Introduction

The program *TransDatRO_code_source* represents the final module of the program TransDatRO which provides, based on some distortion grids, coordinate transformation from European Terrestrial Reference System 1989 (ETRS89) in national reference systems Krasovski 1942 (S-42) with Stereographic Projection 1970 (for the whole territory of Romania), Hayford 1910 with Stereographic Projection 1930 (for the city of Bucharest) and in normal heights system Black Sea 1975.

The source code provides a model for the software developers which can be adapted and implemented both in GNSS receivers for RTK determinations and in Geographic Information Systems (GIS) for large scale spatial data representations. Also, the algorithm presented below clarifies the aspects regarding the particularities of conversions and transformations used in the program.

The executable form of the program (TransDatRO_code_source.jar) can be launched through the following command line (if the program package in the folder *TransDatRO_code_source* was unzipped/copied in the folder C:\Projects):

```
java -jar "C:\Projects\TransDatRO_code_source\dist\TransDatRO_code_source.jar"
```

In this case, the source code of the program *TransDatRO_code_source* can be find in the next text file:

```
C:\Projects\TransDatRO_code_source\src\transdatro_code_source\Main.txt
```

Important note: *The grid with the anomalies of the altitude EGG97_QGRJ.-0002 (EGG97_QGR.GRT) will be updated periodically and progressively according to the evolution of gravimetric determinations that will be carried out in the whole territory of the country.*

Direct and reverse transformation algorithm of the coordinates (B, L, h_{el}) from european system ETRS89 to coordinates (X, Y, H_{MN75}) in Stereographic 1970 (1930) projection and the normal height system Black Sea 1975

I. *Transformation of coordinates (B, L, h_{el}) from european system ETRS89 to coordonates (X, Y, H_{MN75}) in Stereographic 1970 (1930) projection and normal height system Black Sea 1975*

This transformation is presented schematically as follows:

$$(B, L)_{ETRS89} \Rightarrow [1] \Rightarrow (X, Y)_{ObliqueStereographic\_GRS80} \Rightarrow [2] \Rightarrow (X', Y')_{Stereografic1970} \Rightarrow [3] \Rightarrow$$
$$\Rightarrow (X, Y)_{Stereografic1970};$$
$$(h_{el})_{ETRS89} \Rightarrow [4] \Rightarrow (H_{MN75}).$$

Where:

[1] Conversion from ellipsoidal coordinates in ETRS89 (GRS80 ellipsoid) to rectangular coordinates in oblique stereographic projection from the GRS80 ellipsoid (see Appendix 1);

[2] 4 parameter Helmert transformation from rectangular coordinates in oblique stereographic projection from the GRS80 ellipsoid to transformed rectangular coordinates in Stereographic 1970 projection (see Appendix 2);
[3] The interpolation of corrections (distortions) from distortion grid in binary file ETRS89_KRASOVSKI42_2DJ.GRD (the corresponding text file is ETRS89_KRASOVSKI42_2D.GRT) and obtaining the rectangular coordinates in Stereographic 1970 projection by summing the corrections to the coordinates transformed from the previous step (see Appendix 3);
[4] The interpolation of quasigeoid anomalies correspondent to the Black Sea 1975 height system from anomalies grid in EGG97_QGRJ.GRD file (the coresponding text file is EGG97_QGR.GRT) and obtaining the normal heights in Black Sea 1975 system by subtracting the anomalies from ellipsoidal heights in the ETRS89 system (see Appendix 4).

II. ***Transformation of coordinates (X, Y, H_{MN75}) from Stereographic 1970 (1930) projection and normal height system Black Sea 1975 to coordinates (B, L, h_{el}) in european system ETRS89***

Similar to point I, there is also represented the reverse transformation, considering the same steps as before:

$$(X,Y)_{Stereografic1970} \Rightarrow [3'] \Rightarrow (X',Y')_{Stereografic1970} \Rightarrow [2'] \Rightarrow (X,Y)_{ObliqueStereographic\_GRS80} \Rightarrow [1'] \Rightarrow$$
$$\Rightarrow (B,L)_{ETRS89};$$
$$(H_{MN75}) \Rightarrow [4'] \Rightarrow (h_{el})_{ETRS89}.$$

Where:
[3’] The interpolation of corrections (distortions) from distortion grid in ETRS89_KRASOVSKI42_2DJ.GRD file and obtaining the transformed rectangular coordinates in Stereographic 1970 projection by subtracting the corrections from rectangular coordinates in Stereographic 1970 projection;
[2’] 4 parameter Helmert transformation from transformed rectangular coordinates in Stereographic 1970 projection to rectangular coordinates in oblique stereographic projection from the GRS80 ellipsoid;
[1’] The conversion from rectangular coordinates from oblique stereographic projection from the GRS80 ellipsoid to ellipsoidal coordinates ETRS89 (GRS80 ellipsoid);
[4’] The interpolation of quasigeoid anomalies correspondent to the Black Sea 1975 height system from anomalies grid in EGG97_QGRJ.GRD file an obtaining the ellipsoidal heights in ETRS89 by summing the anomalies to normal heights in Black Sea 1975 system.

Sets of coordinates for testing the program TransDatRO_code_source

The program must be tested with coordinate sets in both directions of each individual transformation. The differences between the coordinates obtained in the program and those listed in the tables bellow must be smaller or equal to the following values:

- for East, North, h_{el}, H_MN75 <= ± 0.003 meters;
- for φ, λ <= $\pm 0.00003''$.

Test coordinates for transformation ETRS89 to Stereografic 1970 + Marea Neagra 1975

Point	φ_ETRS89	λ_ETRS89	h_ETRS89	North_St70	East_St70	H_MN75
P1	47 42 56.40000	22 28 32.00000	162.000	693771.731	310723.518	122.714
P2	47 58 33.20000	26 53 26.70000	251.000	721361.806	641283.450	217.451
P3	46 03 57.40000	20 40 11.60000	129.000	516470.189	165265.572	86.267
P4	45 05 18.20000	27 42 24.00000	55.000	402327.815	713143.130	22.941
P5	44 26 51.30000	22 54 09.30000	302.000	329703.378	333185.413	260.515
P6	43 44 37.20000	25 13 48.10000	129.000	249343.594	518651.464	89.294
P7	46 14 47.60000	23 50 46.10000	536.000	528076.247	411159.899	494.894
OutsideGrid	43 11 07.00414	23 08 15.35121	0.000	None	None	None
OutsideBorder	47 56 25.22432	20 35 01.23026	0.000	None	None	None

Test coordinates for transformation Stereografic 1970 + Marea Neagra 1975 to ETRS89

Point	North_St70	East_St70	H_MN75	φ_ETRS89	λ_ETRS89	h_ETRS89
P1	693771.731	310723.518	122.714	47 42 56.40000	22 28 32.00000	162.000
P2	721361.806	641283.450	217.451	47 58 33.20000	26 53 26.70000	251.000
P3	516470.189	165265.572	86.267	46 03 57.40000	20 40 11.60000	129.000
P4	402327.815	713143.130	22.941	45 05 18.20000	27 42 24.00000	55.000
P5	329703.378	333185.413	260.515	44 26 51.30000	22 54 09.30000	302.000
P6	249343.594	518651.464	89.294	43 44 37.20000	25 13 48.10000	129.000
P7	528076.247	411159.899	494.894	46 14 47.60000	23 50 46.10000	536.000
OutsideGrid	188993.152	348668.167	0.000	None	None	None
OutsideBorder	725005.421	170257.544	0.000	None	None	None

Test coordinates for transformation ETRS89 to Stereografic 1930 + Marea Neagra 1975

Point	φ_ETRS89	λ_ETRS89	h_ETRS89	East_St30	North_St30	H_MN75
B1	44 33 07.00000	25 56 17.50000	142.000	543496.555	350692.549	106.860
B2	44 19 32.80000	26 12 38.30000	104.000	565397.499	325744.598	67.929
B3	44 24 21.30000	26 06 38.70000	120.000	557350.864	334573.918	84.174
B4	44 30 11.90000	26 12 41.50000	117.000	565268.634	345471.544	81.984
B5	44 21 47.90000	25 50 37.70000	132.000	536113.846	329685.299	95.541
OutsideGrid	44 13 26.43938	26 01 00.75490	0.000	None	None	None
OutsideBorder	44 34 45.88279	25 51 18.00864	0.000	None	None	None

Test coordinates for transformation Stereografic 1930 + Marea Neagra 1975 to ETRS89

Point	East_St30	North_St30	H_MN75	φ_ETRS89	λ_ETRS89	h_ETRS89
B1	543496.555	350692.549	106.860	44 33 07.00000	25 56 17.50000	142.000
B2	565397.499	325744.598	67.929	44 19 32.80000	26 12 38.30000	104.000
B3	557350.864	334573.918	84.174	44 24 21.30000	26 06 38.70000	120.000
B4	565268.634	345471.544	81.984	44 30 11.90000	26 12 41.50000	117.000
B5	536113.846	329685.299	95.541	44 21 47.90000	25 50 37.70000	132.000
OutsideGrid	550029.245	314298.906	0.000	None	None	None
OutsideBorder	536869.164	353703.293	0.000	None	None	None

Note: The values written in red are changed from the previous edition of the program.

APPENDIX 1

Direct and reverse conversion from ellipsoidal coordinates in ETRS89 (GRS80 ellipsoid) to rectangular coordinates in oblique stereographic projection from the GRS80 ellipsoid

A. *Direct conversion* $(B,L)_{ETRS89} \Rightarrow (X,Y)_{ObliqueStereographic\_GRS80}$

We know:

- The pole of projection $Q_0(\varphi_0 = 46^0, \lambda_0 = 25^0)$ which has the plane rectangular coordinates (false) x (North) = 500000 m and y (East) = 500000 m;
- The scale coefficient k_0 = 0,99975 for the coordinate conversion from tangent plane in Q_0 pole in the secant plane parallel to it;
- The parameters of ellipsoid GRS80:
  - Semi-major axis a = 6378137 m;
  - Flattening f = 1:298.257222101;
- The parameters which define the conformal sphere:

$$R = \sqrt{M_0 N_0}$$

$$n = \sqrt{1 + \frac{e^2 \cos^4 \varphi_0}{(1-e^2)}}$$

$$c = \frac{(n + \sin\varphi_0)(1 - \sin\chi_0)}{(n - \sin\varphi_0)(1 + \sin\chi_0)}$$

where:

$$M_0 = \frac{a(1-e^2)}{(1-e^2 \sin^2 \varphi_0)^{3/2}}$$

$$N_0 = \frac{a}{\sqrt{1-e^2 \sin^2 \varphi_0}}$$

$$b = a(1-f)$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\sin\chi_0 = (w_1 - 1)/(w_1 + 1)$$

$$w_1 = (S_1 \cdot S_2^e)^n$$

$$S_1 = (1 + \sin\varphi_0)/(1 - \sin\varphi_0)$$

$$S_2 = \frac{1 - e\sin\varphi_0}{1 + e\sin\varphi_0}$$

- The conformal latitude and longitude of origin, $Q_0(\chi_0, \Lambda_0)$

$$\chi_0 = \arcsin\left(\frac{w_2 - 1}{w_2 + 1}\right), \text{ where } w_2 = c \cdot (S_1 \cdot S_2^e)^n = c \cdot w_1$$

$$\Lambda_0 = \lambda_0$$

We compute:

- The conformal latitude and longitude of a point $P(\chi, \Lambda)$ for which the conversion is made, which has the geodetic coordinates $P(\varphi, \lambda)$

$$\Lambda = n(\lambda - \Lambda_0) + \Lambda_0$$

$$\chi = \arcsin\left(\frac{w - 1}{w + 1}\right)$$

where:

$$w = c \cdot \left(S_a \cdot S_b^e\right)^n$$

$$S_a = \frac{1 + \sin\varphi}{1 - \sin\varphi}$$

$$S_b = \frac{1 - e\sin\varphi}{1 + e\sin\varphi}$$

- The value β

$$\beta = [1 + \sin\chi\sin\chi_0 - \cos\chi\cos\chi_0\cos(\Lambda - \Lambda_0)]$$

- The rectangular coordinates x (N) and y (E) in the plane of the oblique stereographic projection related to GRS80 ellipsoid

$$N = FN + 2 \cdot R \cdot k_0 [\sin\chi\cos\chi_0 - \cos\chi\sin\chi_0\cos(\Lambda - \Lambda_0)] / \beta$$

$$E = FE + 2 \cdot R \cdot k_0 \cos\chi\sin(\Lambda - \Lambda_0) / \beta$$

where:

FN (false North) = 500000

FE (false East) = 500000

B. ***Reverse conversion*** $(X,Y)_{ObliqueStereographic\_GRS80} \Rightarrow (B,L)_{ETRS89}$

The same initial data as in point I are given.

We compute:

- The conformal latitude and longitude of a point $P(\chi,\Lambda)$ for which the conversion is made, which has the rectangular coordinates in the oblique stereographic projection $P(N,E)$

$$\chi = \chi_0 + 2\arctan\left[\frac{(N-FN)-(E-FE)\tan j/2}{2\cdot R\cdot k_0}\right]$$

$$\Lambda = j + 2i + \Lambda_0$$

where:

$$g = 2\cdot R\cdot k_0 \tan(\pi/4 - \chi_0/2)$$

$$h = 4\cdot R\cdot k_0 \tan\chi_0 + g$$

$$i = \arctan\left(\frac{E-FE}{h+(N-FN)}\right)$$

$$j = \arctan\left(\frac{E-FE}{g-(N-FN)}\right) - i$$

- Geodetic longitude λ

$$\lambda = \Lambda_0 + (\Lambda - \Lambda_0)/n$$

- Isometric latitude ψ

$$\psi = \left(0.5\ln\left[\frac{1+\sin\chi}{c(1-\sin\chi)}\right]\right)/n$$

- Geodetic latitude φ

The first approximation of geodetic latitude is given by the following relation:

$\varphi_1 = 2\arctan(e^{\psi}) - \pi/2$, in which e is the base of natural logarithms

The isometric latitude ψ_i in iteration i corresponding to geodetic latitude φ_i is:

$$\psi_i = \ln\left[\tan(\varphi_i/2+\pi/4)\left(\frac{1-e\sin\varphi_i}{1+e\sin\varphi_i}\right)^{e/2}\right]$$

The geodetic latitude is calculated iteratively with the relation

$$\varphi_{i+1} = \varphi_i - (\psi_i - \psi)\cos\varphi_i (1 - e^2 \sin^2 \varphi_i)/(1 - e^2)$$

until $\varphi_{i+1} - \varphi_i = small \quad enough \quad \cong 0.000001''$.

APPENDIX 2

4 parameter direct and reverse Helmert transformation from rectangular coordinates in oblique stereographic projection from the GRS80 ellipsoid to transformed rectangular coordinates in Stereographic 1970 projection

A. ***Direct transformation*** $(X,Y)_{ObliqueStereographic\_GRS80} \Rightarrow (X',Y')_{Stereografic1970}$

The parameters of the Helmert 2D transformation are given as constant values in the program TransDatRO_code_source.

We have to compute the coordinates transformed into the national Stereographic projection 1970, using the following relations :

$$X' = X_0 + X * m * \cos R_Z - Y * m * \sin R_Z$$
$$Y' = Y_0 + X * m * \sin R_Z + Y * m * \cos R_Z$$

where X=East and Y=North, m=scale coefficient, R_Z = rotation around Z axis and X_0= East translation and Y_0= North translation.

B. ***Reverse transformation*** $(X',Y')_{Stereographic1970} \Rightarrow (X,Y)_{ObliqueStereographic\_GRS80}$

The parameters of the Helmert 2D transformation are given as constant values in the program TransDatRO_code_source.

We have to compute the coordinates in the oblique stereographic projection from the GRS80 ellipsoid using the following relations:

$$X = X'_0 + X' * m' * \cos R'_Z - Y' * m' * \sin R'_Z$$
$$Y = Y'_0 + X' * m' * \sin R'_Z + Y' * m' * \cos R'_Z$$

where X'=East and Y'=North, m'=scale coefficient, R'_Z=rotation around Z' axis and X'_0=East Translation and Y'_0=North translation.

APPENDIX 3

Interpolation of corrections (distortions) from distortion grid ETRS89_KRASOVSKI42_2DJ.GRD **and obtaining the transformed rectangular coordinates,** $(X',Y')_{Stereografic1970} \Rightarrow (X,Y)_{Stereografic1970}$

Distortion interpolation is performed separately for each coordinate (separately on X and Y)

We have:

- The distortion grid as a text file (based on which the corresponding binary file is generated) which has the following structure :

SUBGRID: SISTEMUL 1: ETRS89 → SISTEMUL 2: Krasovski42
GRID PARINTE: NU
CREAT: 04/07/2006
ACTUALIZAT: 01/04/2009
Minimum East (minE):
109783.040
Maximum East (maxE):
904783.040
Minimum North (minN):
213634.564
Maximum North (maxN):
783634.564
East grid interval (stepE):
15000.000
North grid interval (stepN):
15000.000
Number of grid shift values (rows x columns):
2106
Number of dimensions (2 for dEast and dNorth - grid shift values):
2
Grid shift values (dEast dNorth) (columns: minE-->maxE; rows: minN-->maxN):
999.000000 999.000000
999.000000 999.000000
999.000000 999.000000
…
-0.218430 1.274732
-0.203549 0.803709
-0.204519 0.181353
…

The distortions in the grid nodes are written successive in the sequence dEast1, dNorth1, dEast2, dNorth2, …, starting with the bottom left corner of the grid and continuing on each row from left to right and from bottom to top to the top right corner of the grid.

The value 999.000000 means that the node of the grid is outside the Romanian border, and its value is tested by the program to signal to the user that the interpolation point is not in the useful space of transformation.

Based on this text file, the corresponding binary file with direct access to each grid node is generated.

- Relations for interpolation of grid distortions

This algorithm uses the bicubic-spline interpolation whose main feature is the use of smooth interpolation surfaces.

We consider a large grid cell, composed of 16 nodes, represented in the figure 3:

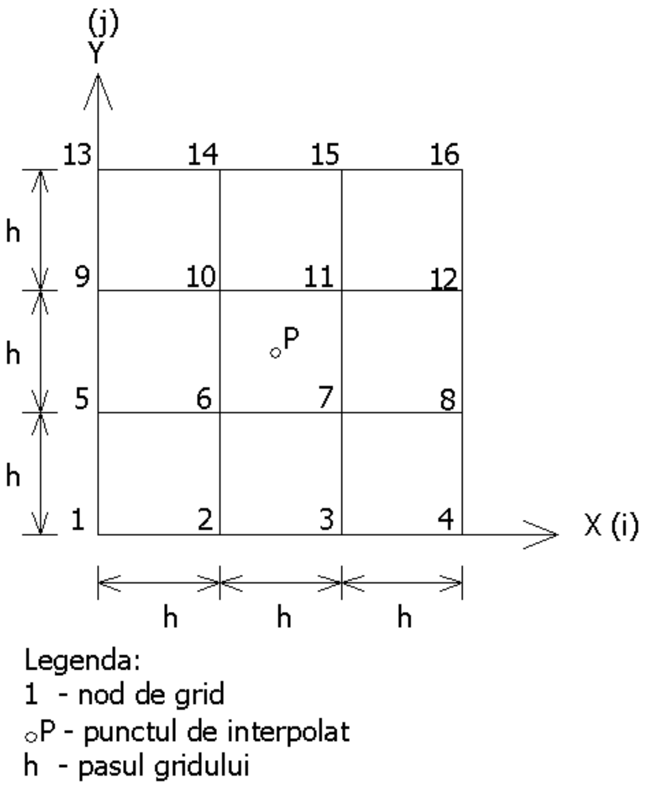

Fig. 3. Composed cell (with 16 nodes) for the interpolation of point P

For interpolation, the following relation is used to describe a bicubic-spline surface:

$$p(x,y)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}x^{i}y^{j}$$

We consider the function, $f(x,y)=p(x,y)$ which has known the partial derivatives f_x, f_y and f_{xy} known in the corners of the unit square (side h = 1) defined by points 6, 7, 11, 10 which have the coordinates: 6(0,0), 7(1,0), 10(0,1) şi 11(1,1).

The a_{ij} coefficients are determined from the following system of condition equations of the function f :

1. $f(0,0) = p(0,0) = a_{00}$
2. $f(1,0) = p(1,0) = a_{00} + a_{10} + a_{20} + a_{30}$
3. $f(0,1) = p(0,1) = a_{00} + a_{01} + a_{02} + a_{03}$
4. $f(1,1) = p(1,1) = \sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}$
5. $f_x(0,0) = p_x(0,0) = a_{10}$
6. $f_x(1,0) = p_x(1,0) = a_{10} + 2a_{20} + 3a_{30}$
7. $f_x(0,1) = p_x(0,1) = a_{10} + a_{11} + a_{12} + a_{13}$
8. $f_x(1,1) = p_x(1,1) = \sum_{i=1}^{3}\sum_{j=0}^{3}a_{ij}i$
9. $f_y(0,0) = p_y(0,0) = a_{01}$
10. $f_y(1,0) = p_y(1,0) = a_{01} + a_{11} + a_{21} + a_{31}$
11. $f_y(0,1) = p_y(0,1) = a_{01} + 2a_{02} + 3a_{03}$
12. $f_y(1,1) = p_y(1,1) = \sum_{i=0}^{3}\sum_{j=1}^{3}a_{ij}j$
13. $f_{xy}(0,0) = p_{xy}(0,0) = a_{11}$
14. $f_{xy}(1,0) = p_{xy}(1,0) = a_{11} + 2a_{21} + 3a_{31}$
15. $f_{xy}(0,1) = p_{xy}(0,1) = a_{11} + 2a_{12} + 3a_{13}$

16. $f_{xy}(1,1) = p_{xy}(1,1) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} ij$

Where the expressions p_x, p_y, p_{xy} are computed by following identities:

$$p_x(x,y) = \sum_{i=1}^{3}\sum_{j=0}^{3} a_{ij} i x^{i-1} y^{j}$$

$$p_y(x,y) = \sum_{i=0}^{3}\sum_{j=1}^{3} a_{ij} x^{i} j y^{j-1}$$

$$p_{xy}(x,y) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} i x^{i-1} j y^{j-1}$$

Derivatives values f_x, f_y, f_{xy} are computed in the unit square 6, 7, 11, 10 with the help of neighboring node values, using the finite difference method.

For each corner of the unit square we consider separately a local system with origin (i, j) in the considered node, as in figure no. 4 :

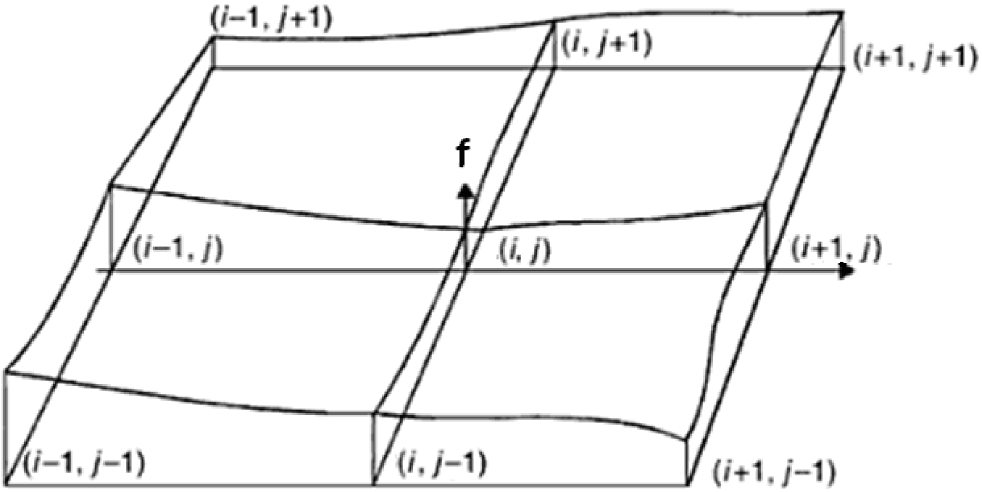

Fig. 4. Relative coordinate system (i, j) for derivatives calculation

Derivatives of function f are computed by the following relations:

$$f_x^{d} = \frac{-p(i+2,j) + 4*p(i+1,j) - 3*p(i,j)}{2}$$

$$f_x^{i} = \frac{3*p(i,j) - 4*p(i-1,j) + p(i-2,j)}{2}$$

$$f_y^{d} = \frac{-p(i,j+2) + 4*p(i,j+1) - 3*p(i,j)}{2}$$

$$f_y^{i} = \frac{3*p(i,j) - 4*p(i,j-1) + p(i,j-2)}{2}$$

$$f_{xy}^{m} = \frac{p(i-1,j-1) + p(i+2,j+2) - p(i+1,j-1) - p(i-1,j+1)}{4}$$

We have to compute:

- Predicted distorsion $p(x, y)$ in a new point P;
- Final coordinates corrected by such kind of relation $X = X' + p(x, y)$

Similarly, the correction for the Y coordinate is also interpolated, using the same relation of bicubic-spline surface but with other computed coefficients which depend on Y distorsions around the interpolated point.

APPENDIX 4

Interpolation of anomalies of the quasigeoid corespondent of Black Sea 1975 height system with anomalies from file EGG97_QGRJ.GRD and obtaining of the normal heights in Black Sea 1975 system, $(h_{el})_{ETRS89} \Rightarrow (H_{MN75})$.

We have:

- The distorsion grid as a text file (based on which the corresponding binary file EGG97_QGRJ.GRD is generated) which has the following structure:

```
SUBGRID: EGG97 - QGeoid Romania
GRID PARINTE: NU
CREAT: 14/11/2007
ACTUALIZAT: 09/07/2009
Minimum East (minE):
   19.945609
Maximum East (maxE):
   30.057000
Minimum North (minN):
   43.421744
Maximum North (maxN):
   48.533000
East grid interval (stepE):
    0.111111113
North grid interval (stepN):
    0.111111113
Number of grid shift values (rows x columns):
4324
Number of dimensions (1 for dZita - grid shift value):
1
Grid shift values (dEast dNorth) (columns: minE-->maxE; rows: minN-->maxN):
 999.000000
 999.000000
 999.000000
...
  41.977842
  41.784944
  41.561786
...
```

We compute:

- The predicted anomaly $z(\varphi,\lambda)$ in a new point P as in Appendix 3;
- The final height with such kind a relation $H_{MN75} = h_{el} - z(\varphi,\lambda)$